

Weekly Review 1

① Find the limits algebraically

$$\textcircled{a} \lim_{x \rightarrow -3} \frac{x^2 - 3x - 18}{x + 3}$$

$$\textcircled{b} \lim_{x \rightarrow 0} \frac{x + \sin x}{x}$$

$$\textcircled{c} \lim_{x \rightarrow 0} \frac{3 \sin^4 x}{\sin 3x}$$

$$\textcircled{d} \lim_{x \rightarrow 3} \frac{2x^3 - 4x^2 - 18}{x - 3}$$

- ② Find all the discontinuities (if any) of $f(x)$ and label these removable or nonremovable. If removable, give an extended function that is continuous at the given point. Fully justify all your statements.

$$\textcircled{a} \quad f(x) = \begin{cases} 3-x, & x < 2 \\ \frac{x}{2} + 1, & x > 2 \end{cases}$$

$$\textcircled{b} \quad f(x) = \begin{cases} \frac{1}{x-1}, & x < 1 \\ x^3 - 2x + 5, & x \geq 1 \end{cases}$$

- ③ Find $\frac{d}{dx} f(x)$ using both the difference quotient and the rules for derivatives from sect. 3.3.

$$f(x) = \frac{2x^2 + 1}{x + 2}$$

- ④ Find the equations for both the tangent line and normal line to $f(x) = x^2 - 3x - 1$ at $x = 0$.

- ⑤ What is the rate of change of the volume of a cone with respect to the radius when the radius is 4 inches. The height of the cone is equal to the diameter and $V_{\text{cone}} = \frac{1}{3} \pi r^2 h$.

Answers to Weekly Review 1

① (a) -9 (b) 2 (c) 4 (d) 30

② (a) A non-removable jump discontinuity at $x=2$. One-sided limits are different

(b) Non-removable infinite discontinuity at $x=1$, one-sided limits are different

③ $f'(x) = \frac{2x^2 + 8x - 1}{x^2 + 4x + 4}$

④ Tangent Line: $y = -3x - 1$
Normal Line: $y = \frac{1}{3}x - 1$

⑤ 32π